7.9 Plasma physics

Warm plasmas

Landau length	$$l_L = \frac{e^2}{4\pi\epsilon_0 k_B T_e} \quad (7.248)$$ $$\simeq 1.67\times10^{-5}T_e^{-1} \quad \text{m} \quad (7.249)$$	l_L Landau length $-e$ electronic charge ϵ_0 permittivity of free space k_B Boltzmann constant T_e electron temperature (K)
Electron Debye length	$$\lambda_{De} = \left(\frac{\epsilon_0 k_B T_e}{n_e e^2}\right)^{1/2} \quad (7.250)$$ $$\simeq 69(T_e/n_e)^{1/2} \quad \text{m} \quad (7.251)$$	λ_{De} electron Debye length n_e electron number density (m^{-3})
Debye screening[a]	$$\phi(r) = \frac{q\exp(-2^{1/2}r/\lambda_{De})}{4\pi\epsilon_0 r} \quad (7.252)$$	ϕ effective potential q point charge r distance from q
Debye number	$$N_{De} = \frac{4}{3}\pi n_e \lambda_{De}^3 \quad (7.253)$$	N_{De} electron Debye number
Relaxation times ($B=0$)[b]	$$\tau_e = 3.44\times10^5 \frac{T_e^{3/2}}{n_e \ln\Lambda} \quad \text{s} \quad (7.254)$$ $$\tau_i = 2.09\times10^7 \frac{T_i^{3/2}}{n_e \ln\Lambda}\left(\frac{m_i}{m_p}\right)^{1/2} \quad \text{s} \quad (7.255)$$	τ_e electron relaxation time τ_i ion relaxation time T_i ion temperature (K) $\ln\Lambda$ Coulomb logarithm (typically 10 to 20) B magnetic flux density
Characteristic electron thermal speed[c]	$$v_{te} = \left(\frac{2k_B T_e}{m_e}\right)^{1/2} \quad (7.256)$$ $$\simeq 5.51\times10^3 T_e^{1/2} \quad \text{m s}^{-1} \quad (7.257)$$	v_{te} electron thermal speed m_e electron mass

[a] Effective (Yukawa) potential from a point charge q immersed in a plasma.
[b] Collision times for electrons and *singly* ionised ions with Maxwellian speed distributions, $T_i \lesssim T_e$. The Spitzer conductivity can be calculated from Equation (7.233).
[c] Defined so that the Maxwellian velocity distribution $\propto \exp(-v^2/v_{te}^2)$. There are other definitions (see *Maxwell–Boltzmann distribution* on page 112).

Electromagnetic propagation in cold plasmas[a]

Plasma frequency	$(2\pi\nu_{\mathrm{p}})^2=\frac{n_{\mathrm{e}}e^2}{\epsilon_0 m_{\mathrm{e}}}=\omega_{\mathrm{p}}^2$ (7.258) $\nu_{\mathrm{p}}\simeq 8.98 n_{\mathrm{e}}^{1/2}$ Hz (7.259)		ν_{p} plasma frequency ω_{p} plasma angular frequency n_{e} electron number density (m^{-3}) m_{e} electron mass
Plasma refractive index ($B=0$)	$\eta=\left[1-(\nu_{\mathrm{p}}/\nu)^2\right]^{1/2}$	(7.260)	$-e$ electronic charge ϵ_0 permittivity of free space η refractive index ν frequency
Plasma dispersion relation ($B=0$)	$c^2k^2=\omega^2-\omega_{\mathrm{p}}^2$	(7.261)	k wavenumber ($=2\pi/\lambda$) ω angular frequency ($=2\pi/\nu$) c speed of light
Plasma phase velocity ($B=0$)	$v_\phi=c/\eta$	(7.262)	v_ϕ phase velocity
Plasma group velocity ($B=0$)	$v_{\mathrm{g}}=c\eta$ (7.263) $v_\phi v_{\mathrm{g}}=c^2$ (7.264)		v_{g} group velocity
Cyclotron (Larmor, or gyro-) frequency	$2\pi\nu_{\mathrm{C}}=\frac{qB}{m}=\omega_{\mathrm{C}}$ (7.265) $\nu_{\mathrm{Ce}}\simeq 28\times10^9 B$ Hz (7.266) $\nu_{\mathrm{Cp}}\simeq 15.2\times10^6 B$ Hz (7.267)		ν_{C} cyclotron frequency ω_{C} cyclotron angular frequency ν_{Ce} electron ν_{C} ν_{Cp} proton ν_{C} q particle charge B magnetic flux density (T)
Larmor (cyclotron, or gyro-) radius	$r_{\mathrm{L}}=\frac{v_\perp}{\omega_{\mathrm{C}}}=v_\perp\frac{m}{qB}$ (7.268) $r_{\mathrm{Le}}=5.69\times10^{-12}\left(\frac{v_\perp}{B}\right)$ m (7.269) $r_{\mathrm{Lp}}=10.4\times10^{-9}\left(\frac{v_\perp}{B}\right)$ m (7.270)		m particle mass (γm if relativistic) r_{L} Larmor radius r_{Le} electron r_{L} r_{Lp} proton r_{L} $v_\perp$ speed $\perp$ to $\boldsymbol{B}$ ($\mathrm{m\,s^{-1}}$)
Mixed propagation modes[b]	$\eta^2=1-\frac{X(1-X)}{(1-X)-\frac{1}{2}Y^2\sin^2\theta_B\pm S}$, (7.271) where $X=(\omega_{\mathrm{p}}/\omega)^2$, $Y=\omega_{\mathrm{Ce}}/\omega$, and $S^2=\frac{1}{4}Y^4\sin^4\theta_B+Y^2(1-X)^2\cos^2\theta_B$		θ_B angle between wavefront normal ($\hat{\boldsymbol{k}}$) and $\boldsymbol{B}$
Faraday rotation[c]	$\Delta\psi=\underbrace{\frac{\mu_0 e^3}{8\pi^2 m_{\mathrm{e}}^2 c}}_{2.63\times10^{-13}}\lambda^2\int_{\mathrm{line}} n_{\mathrm{e}}\boldsymbol{B}\cdot\mathrm{d}\boldsymbol{l}$ (7.272) $=R\lambda^2$ (7.273)		$\Delta\psi$ rotation angle λ wavelength ($=2\pi/k$) $\mathrm{d}\boldsymbol{l}$ line element in direction of wave propagation R rotation measure

[a] I.e., plasmas in which electromagnetic force terms dominate over thermal pressure terms. Also taking $\mu_{\mathrm{r}}=1$.
[b] In a collisionless electron plasma. The ordinary and extraordinary modes are the $+$ and $-$ roots of S^2 when $\theta_B=\pi/2$. When $\theta_B=0$, these roots are the right and left circularly polarised modes respectively, using the optical convention for handedness.
[c] In a tenuous plasma, SI units throughout. $\Delta\psi$ is taken positive if $\boldsymbol{B}$ is directed towards the observer.

Magnetohydrodynamics[a]

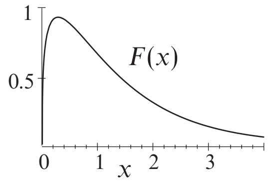
Sound speed	$v_s = \left(\frac{\gamma p}{\rho}\right)^{1/2} = \left(\frac{2\gamma k_B T}{m_p}\right)^{1/2}$ (7.274) $\simeq 166 T^{1/2}\,\mathrm{m\,s^{-1}}$ (7.275)	v_s sound (wave) speed γ ratio of heat capacities p hydrostatic pressure ρ plasma mass density k_B Boltzmann constant T temperature (K) m_p proton mass
Alfvén speed	$v_A = \frac{B}{(\mu_0 \rho)^{1/2}}$ (7.276) $\simeq 2.18 \times 10^{16} B n_e^{-1/2} \quad \mathrm{m\,s^{-1}}$ (7.277)	v_A Alfvén speed B magnetic flux density (T) μ_0 permeability of free space n_e electron number density ($\mathrm{m^{-3}}$)
Plasma beta	$\beta = \frac{2\mu_0 p}{B^2} = \frac{4\mu_0 n_e k_B T}{B^2} = \frac{2v_s^2}{\gamma v_A^2}$ (7.278)	β plasma beta (ratio of hydrostatic to magnetic pressure)
Direct electrical conductivity	$\sigma_d = \frac{n_e^2 e^2 \sigma}{n_e^2 e^2 + \sigma^2 B^2}$ (7.279)	$-e$ electronic charge σ_d direct conductivity σ conductivity ($B = 0$)
Hall electrical conductivity	$\sigma_H = \frac{\sigma B}{n_e e}\sigma_d$ (7.280)	σ_H Hall conductivity
Generalised Ohm's law	$\boldsymbol{J} = \sigma_d(\boldsymbol{E} + \boldsymbol{v}\times\boldsymbol{B}) + \sigma_H \hat{\boldsymbol{B}}\times(\boldsymbol{E} + \boldsymbol{v}\times\boldsymbol{B})$ (7.281)	$\boldsymbol{J}$ current density $\boldsymbol{E}$ electric field $\boldsymbol{v}$ plasma velocity field $\hat{\boldsymbol{B}}$ $= \boldsymbol{B}/\lvert\boldsymbol{B}\rvert$
Resistive MHD equations (single-fluid model)[b]	$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla\times(\boldsymbol{v}\times\boldsymbol{B}) + \eta\nabla^2\boldsymbol{B}$ (7.282) $\frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v}\cdot\nabla)\boldsymbol{v} = -\frac{\nabla p}{\rho} + \frac{1}{\mu_0\rho}(\nabla\times\boldsymbol{B})\times\boldsymbol{B} + \nu\nabla^2\boldsymbol{v} + \frac{1}{3}\nu\nabla(\nabla\cdot\boldsymbol{v}) + \boldsymbol{g}$ (7.283)	μ_0 permeability of free space η magnetic diffusivity [$= 1/(\mu_0\sigma)$] ν kinematic viscosity $\boldsymbol{g}$ gravitational field strength
Shear Alfvénic dispersion relation[c]	$\omega = k v_A \cos\theta_B$ (7.284)	ω angular frequency ($= 2\pi\nu$) $\boldsymbol{k}$ wavevector ($k = 2\pi/\lambda$) θ_B angle between $\boldsymbol{k}$ and $\boldsymbol{B}$
Magnetosonic dispersion relation[d]	$\omega^2 k^2 (v_s^2 + v_A^2) - \omega^4 = v_s^2 v_A^2 k^4 \cos^2\theta_B$ (7.285)	

[a]For a warm, fully ionised, electrically neutral p^+/e^- plasma, $\mu_r = 1$. Relativistic and displacement current effects are assumed to be negligible and all oscillations are taken as being well below all resonance frequencies.
[b]Neglecting bulk (second) viscosity.
[c]Nonresistive, inviscid flow.
[d]Nonresistive, inviscid flow. The greater and lesser solutions for ω^2 are the fast and slow magnetosonic waves respectively.

Synchrotron radiation

Power radiated by a single electron[a]	$P_{\text{tot}} = 2\sigma_{\text{T}} c u_{\text{mag}} \gamma^2 \left(\frac{v}{c}\right)^2 \sin^2\theta$	(7.286)
	$\simeq 1.59 \times 10^{-14} B^2 \gamma^2 \left(\frac{v}{c}\right)^2 \sin^2\theta \quad \text{W}$	(7.287)
... averaged over pitch angles	$P_{\text{tot}} = \frac{4}{3}\sigma_{\text{T}} c u_{\text{mag}} \gamma^2 \left(\frac{v}{c}\right)^2$	(7.288)
	$\simeq 1.06 \times 10^{-14} B^2 \gamma^2 \left(\frac{v}{c}\right)^2 \quad \text{W}$	(7.289)
Single electron emission spectrum[b]	$P(\nu) = \frac{3^{1/2} e^3 B \sin\theta}{4\pi\epsilon_0 c m_{\text{e}}} F(\nu/\nu_{\text{ch}})$	(7.290)
	$\simeq 2.34 \times 10^{-25} B \sin\theta F(\nu/\nu_{\text{ch}}) \quad \text{W Hz}^{-1}$	(7.291)
Characteristic frequency	$\nu_{\text{ch}} = \frac{3}{2}\gamma^2 \frac{eB}{2\pi m_{\text{e}}} \sin\theta$	(7.292)
	$\simeq 4.2 \times 10^{10} \gamma^2 B \sin\theta \quad \text{Hz}$	(7.293)
Spectral function	$F(x) = x \int_x^{\infty} K_{5/3}(y)\,\mathrm{d}y$	(7.294)
	$\simeq \begin{cases} 2.15 x^{1/3} & (x \ll 1) \\ 1.25 x^{1/2} \mathrm{e}^{-x} & (x \gg 1) \end{cases}$	(7.295)

P_{tot}	total radiated power
σ_{T}	Thomson cross section
u_{mag}	magnetic energy density $= B^2/(2\mu_0)$
$\boldsymbol{v}$	electron velocity $(\sim c)$
γ	Lorentz factor $= [1-(v/c)^2]^{-1/2}$
θ	pitch angle (angle between $\boldsymbol{v}$ and $\boldsymbol{B}$)
$\boldsymbol{B}$	magnetic flux density
c	speed of light
$P(\nu)$	emission spectrum
ν	frequency
ν_{ch}	characteristic frequency
$-e$	electronic charge
ϵ_0	free space permittivity
m_{e}	electronic (rest) mass
F	spectral function
$K_{5/3}$	modified Bessel fn. of the 2nd kind, order 5/3



[a]This expression also holds for cyclotron radiation ($v \ll c$).
[b]I.e., total radiated power per unit frequency interval.

Bremsstrahlung[a]

Single electron and ion[b]

$$\frac{\mathrm{d}W}{\mathrm{d}\omega} = \frac{Z^2e^6}{24\pi^4\epsilon_0^3c^3m_e^2}\frac{\omega^2}{\gamma^2v^4}\left[\frac{1}{\gamma^2}K_0^2\left(\frac{\omega b}{\gamma v}\right)+K_1^2\left(\frac{\omega b}{\gamma v}\right)\right] \quad (7.296)$$

$$\simeq \frac{Z^2e^6}{24\pi^4\epsilon_0^3c^3m_e^2b^2v^2} \quad (\omega b \ll \gamma v) \quad (7.297)$$

Thermal bremsstrahlung radiation ($v \ll c$; Maxwellian distribution)

$$\frac{\mathrm{d}P}{\mathrm{d}V\,\mathrm{d}\nu} = 6.8\times 10^{-51}Z^2T^{-1/2}n_in_eg(\nu,T)\exp\left(\frac{-h\nu}{kT}\right) \quad \mathrm{W\,m^{-3}\,Hz^{-1}} \quad (7.298)$$

$$\text{where} \quad g(\nu,T)\simeq\begin{cases}0.28[\ln(4.4\times10^{16}T^3\nu^{-2}Z^{-2})-0.76] & (h\nu\ll kT\lesssim 10^5kZ^2)\\ 0.55\ln(2.1\times10^{10}T\nu^{-1}) & (h\nu \ll 10^5kZ^2\lesssim kT)\\ (2.1\times10^{10}T\nu^{-1})^{-1/2} & (h\nu\gg kT)\end{cases} \quad (7.299)$$

$$\frac{\mathrm{d}P}{\mathrm{d}V}\simeq 1.7\times10^{-40}Z^2T^{1/2}n_in_e \quad \mathrm{W\,m^{-3}} \quad (7.300)$$

ω	angular frequency ($=2\pi\nu$)	v	electron velocity	W	energy radiated
Ze	ionic charge	K_i	modified Bessel functions of order i (see page 47)	T	electron temperature (K)
$-e$	electronic charge	γ	Lorentz factor $=[1-(v/c)^2]^{-1/2}$	n_i	ion number density ($\mathrm{m^{-3}}$)
ϵ_0	permittivity of free space	P	power radiated	n_e	electron number density ($\mathrm{m^{-3}}$)
c	speed of light	V	volume	k	Boltzmann constant
m_e	electronic mass	ν	frequency (Hz)	h	Planck constant
b	collision parameter[c]			g	Gaunt factor

[a]Classical treatment. The ions are at rest, and all frequencies are above the plasma frequency.
[b]The spectrum is approximately flat at low frequencies and drops exponentially at frequencies $\gtrsim \gamma v/b$.
[c]Distance of closest approach.